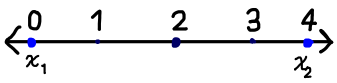
WMW #4: What to do if you believe in patterns?

To get started, we are going to take two objects: an orange, and a 20kg mass.

Question: Do these two objects fall to the ground at the same time?
YES. Here's why: The acceleration due to gravity is $9.81\,m/s^2$, regardless of mass.

Let's review some kinematics:
Position (x): where the object is



Velocity (v): how fast the object is, and in what direction.

- v=0 not moving
- v=1 slowly moving
- v=3 quickly moving
- v= -1 moves backwards

Acceleration: the change of speed

- a=0 constant speed
- a=2 goes faster
- a=-2 goes slower (and eventually goes backwards)

We see some patterns here...
You can calculate the distance using a formula:

$$x(t) = x_0 + v_0 t + \frac{at^2}{2!} + \frac{bt^3}{3!} + \frac{ct^4}{4!} + \frac{dt^5}{5!} (\dots)$$

(×2) (×3) (×4) (×5) (×n+1)

See the patterns?

There are, indeed lots of patterns.

1. 1, 3, 5, 7, 9, 11, 13, 15 (+2 +2 +2 +2 +2 +2 +2)
2. 3, 6, 12, 24, 48, 96 (x2 x2 x2 x2 x2)
3. 2, 5, 10, 17, 26, 37, 50 (+3 +5 +7 +9 +11 +13; +2 +2 +2 +2 +2)
4. 5, 13, 35, 97, 275 (+8 +22 +62 +178; +14 +40 +116; +26 +76) No... this is a geometric pattern...

Note: With diagonals, you can make "polynomial patterns."

```
\2  7  16  29  46  67
 \5  9  13  17  21
  \4  4   4   4   ← always constant
```

Theorem: If you know the numbers on the diagonal, then you know the rest of the pattern.

Remember this pattern?

```
1 \2  7  16  29  46  67
 1 \5  9  13  17  21
  4 \4  4   4   4
```

You can determine the formula based on the patterns you see.

Another one:

```
5 \8  7  4  4  15  48  117  239
 3 \-1 -3  0  11  33  69  122
 -4 \-2  3  11  22  36  53
   2 \5  8  11  14  17
    3 \3  3  3  3
```

~~To find the formula,
1. [3, 2, -4, 3, 5]
2. $\frac{3x^4}{24} + \frac{2x^3}{6} - \frac{4x^2}{2} + 3x + 5$
$\frac{x^4}{8} + \frac{x^3}{3} - 2x^2 + 3x + 5$~~

No. Not quite.

If you want to find the formula, you will have to interpolate functions. This is part of numerical analysis. (MAT2630)

OK. Let's take one step back.
Let's take the diagonal of common functions.

1
```
\1  1  1  1
 \0  0  0
```

n
```
\1  2  3  4
 \1  1  1
  \0  0
```

n^2
```
\1  4  9  16  25
 \3  5  7  9
  \2  2  2
   \0  0
```

n^3
```
\1  8  27  64  125
 \7  19  37  61
  \12  18  24
   \6  6
    \0
```

n^4
```
\1  16  81  256  625  1,296
 \15  65  175  369  671
  \50  110  194  302
   \60  84  108
    \24  24
     \0
```

$\frac{n^2+n}{2}$
```
\1  3  6  10  15
 \2  3  4  5
  \1  1  1
   \0  0
```

You can use the diagonal numbers to find formulas.
Example:

```
\2  7  16  29  46  67
 \5  9  13  17  21
  \4  4   4   4
```

1	n	n^2	n^3 (∅)	n^4 (∅)		EQ		$2n^2$	$-n$	1
1	1	1	1	1		2		2	-1	1
0	1	3	7	15	—	5		6	-1	0
0	0	2	12	50		4		4	0	0
0	0	0	6	60	—	0		0	0	0
0	0	0	0	24		0		0	0	0

Solution: $2n^2 - n + 1$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1

Activity: Pascal's Triangle has lots of patterns. What patterns do you notice?