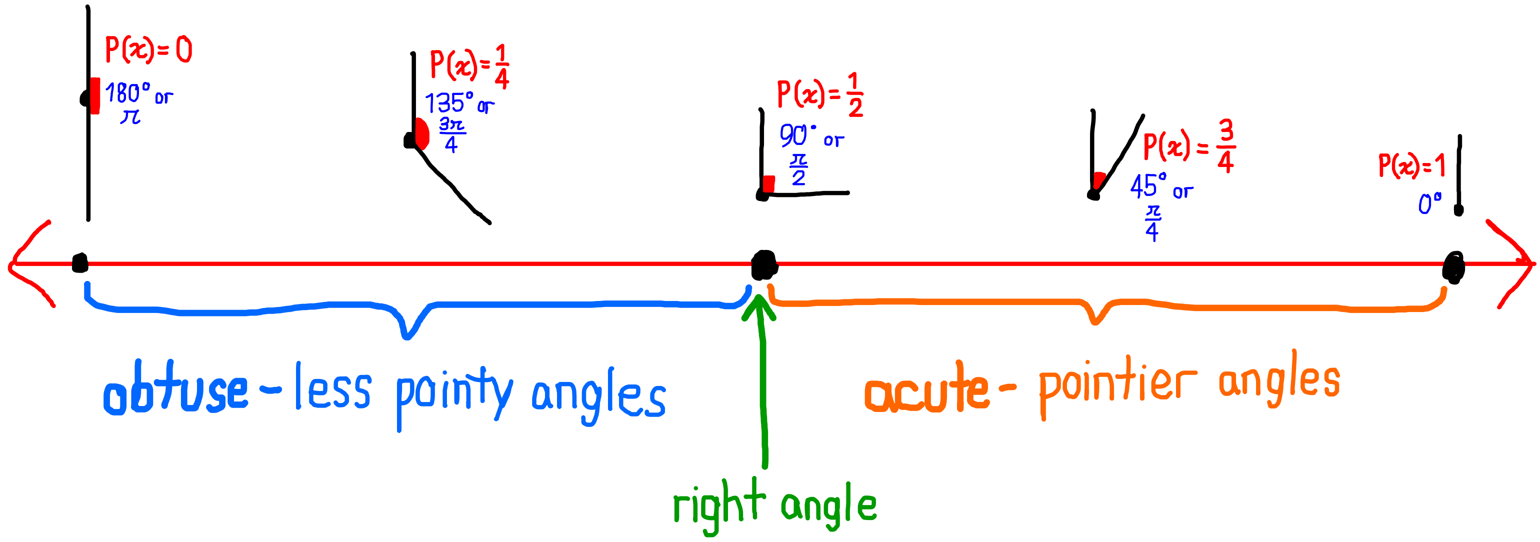
WMW #1 - How Pointy Is a Polygon?

First, we determine the pointiness of an angle.



The general formula for pointiness of angle $P(x) = \left[\dfrac{180 - a}{180}\right]$

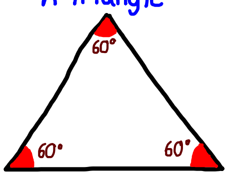
(assuming degrees)

The pointiness of any polygon is $\left(P(x^\circ) = \frac{2}{n}\right)$ times n, which represents the # of angles.

Why is the pointiness always **2**?

Let's demonstrate:

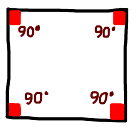
A triangle



$P(60^\circ) = \frac{2}{3}$

$3P(60^\circ) = 2$

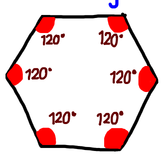
A square



$P(90^\circ) = \frac{1}{2}$

$4P(90^\circ) = 2$

A hexagon



$P(120^\circ) = \frac{1}{3}$

$6P(120^\circ) = 2$

A 48-gon?!

We will find the interior angles.

Interior angle formula:

48-gon: $\dfrac{180^\circ(n-2)}{n}$

let $n = 48$

$= \dfrac{180(48-2)}{48} = \dfrac{180(46)}{48} = 180 \times \dfrac{46}{48}$

$= 180 \times \dfrac{23}{24} = \dfrac{4140}{24} = 172.5^\circ$

Then, we find the pointiness.

$\dfrac{180 - 172.5}{180} = \dfrac{7.5}{180}$

$= \dfrac{1}{24}$ ← *not very pointy!*

Then, sides × pointiness.

$48\left(\frac{1}{24}\right) = 2$

$48P(172.5^\circ) = 2$

A bigon (2-gon) may not be a polygon, but...

still has pointiness of two! Why?!

←0°

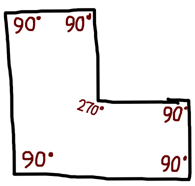
$P(0^\circ) = 1$

$P(0^\circ) = +1$

←0°

2

A convex polygon

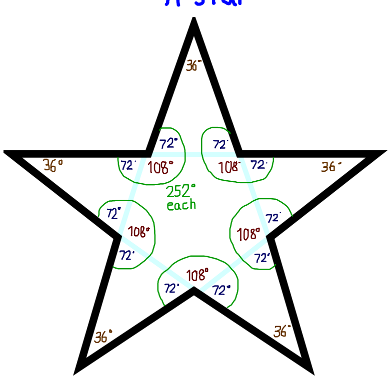


$P(270^\circ) = -\frac{1}{2}$ ← negative pointiness

$5P(90^\circ) = 2\frac{1}{2}$

$P(270^\circ) + 5P(90^\circ) = \mathbf{2}$

A star



1) Find the interior angles of a pentagon

$= \dfrac{180(5-2)}{5} = \dfrac{180(3)}{5} = \dfrac{540}{5} = 108^\circ$

2) Find the pointiness of $P(252^\circ)$ and $P(36^\circ)$

$P(252^\circ) = \dfrac{180 - 252}{180} = -\dfrac{72}{180} = -\dfrac{2}{5}$ ← negative pointiness

$P(36^\circ) = \dfrac{180 - 36}{180} = \dfrac{144}{180} = \dfrac{4}{5}$

3) Find the pointiness of a star!

$5P(36^\circ) = 4$

$5P(252^\circ) = -2$

2